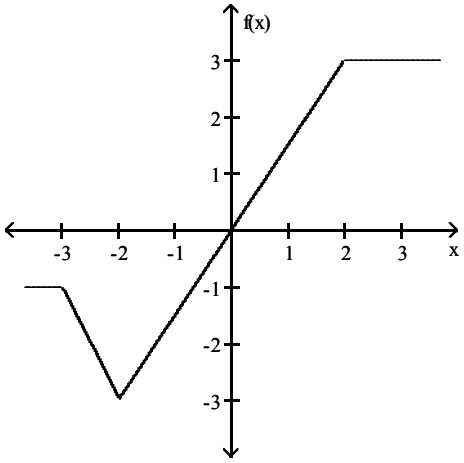
MAC 2311
Chapter 3 Review Materials (Part 1)
Topics Include Max and Min values , Concavity, Curve Sketching (first and second derivative analysis)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

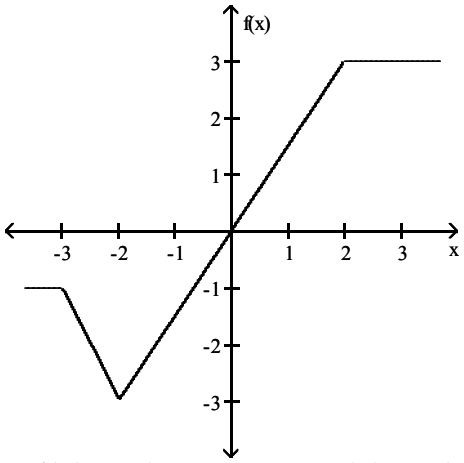
Identify the open intervals where the function is changing as requested.

1) Increasing 1) _____



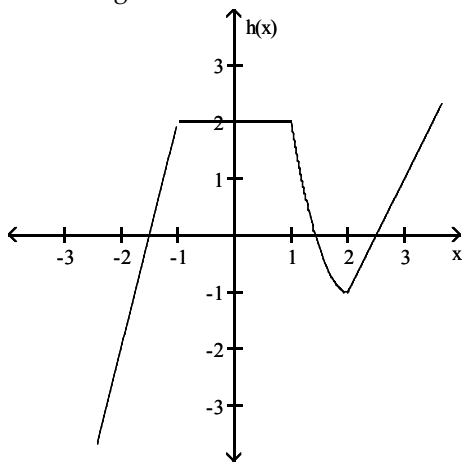
A) (–2, 2) B) (–3, ∞) C) (–2, ∞) D) (–3, 3)

2) Decreasing 2) _____



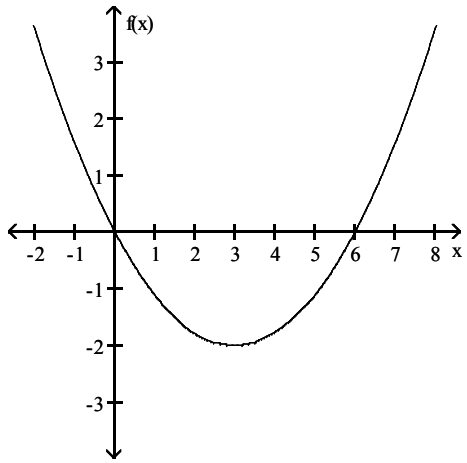
A) (–∞, –3) B) (–∞, –2) C) (–3, –2) D) (0, –2)

3) Increasing 3) _____



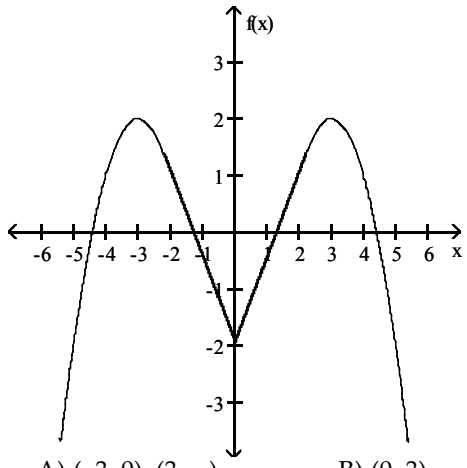
A) (-∞, –1) B) (-∞,–1), (2, ∞) C) (–1, ∞) D) (–1, 2)

4) Decreasing 4) _____



A) (0, –2) B) (–∞, 3) C) (–∞, –2) D) (0, 3)

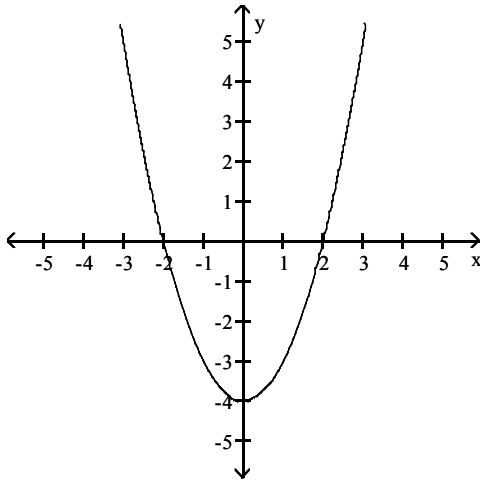
5) Increasing 5) _____



A) (–3, 0), (3, ∞) B) (0, 3) C) (-∞, –3), (0, 3) D) (-∞, –3), (3, ∞)

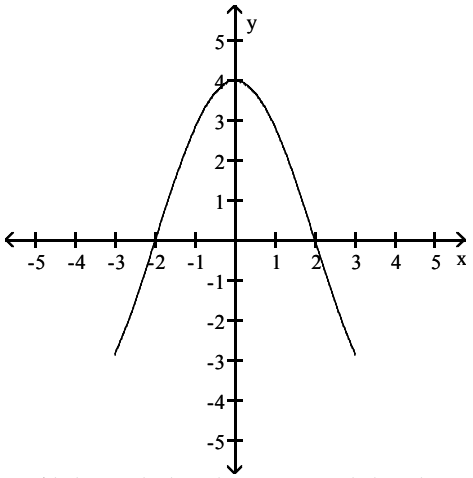
Suppose that the function with the given graph is not f(x), but f′(x). Find the open intervals where f(x) is increasing or decreasing as indicated.

6) Increasing 6) _____



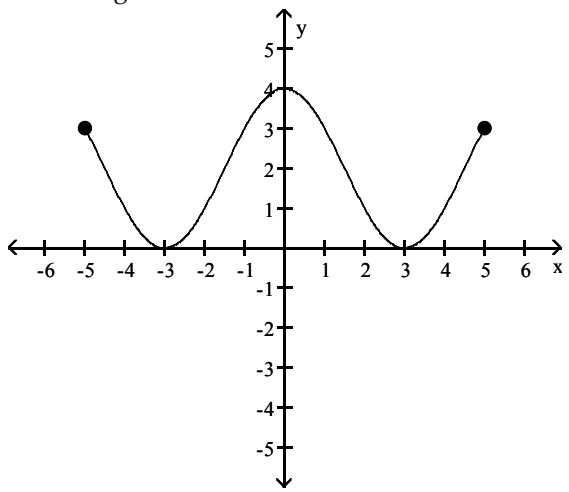
A) (0, ∞) B) (–2, 2) C) (2, ∞) D) (-∞, –2), (2, ∞)

7) Increasing 7) _____



A) (-∞, –2), (2, ∞) B) (0, ∞) C) (2, ∞) D) (–2, 2)

8) Increasing 8) _____



A) $(-\infty, \infty)$ B) $(-3, 3)$
C) $(-\infty, -3), (-3, 3), (3, \infty)$ D) $(-\infty, -3), (3, \infty)$

Find all the critical numbers of the function.

9) $y = 2.5 - 3.9x + 1.2x^2$ 9) _____

A) $\frac{25}{39}$ B) $\frac{7}{12}$ C) $\frac{13}{8}$ D) $-\frac{25}{24}$

10) $f(x) = 2x^3 + 3x^2 - 36x + 8$ 10) _____

A) $-3, 2$ B) -2 C) $3, -2$ D) 6

11) $f(x) = (x + 2)^{1/5}$ 11) _____

A) $\frac{2}{5}$ B) -2 C) 2 D) 10

12) $y = x^{1/5} - x^{6/5}$ 12) _____

A) $-\frac{1}{5}, 0$ B) $0, \frac{1}{6}$ C) $\frac{1}{5}$ D) $\frac{1}{6}$

Find the open interval(s) where the function is changing as requested.

13) Increasing; $y = 7x - 5$ 13) _____

A) $(-\infty, \infty)$ B) $(-\infty, 7)$ C) $(-5, 7)$ D) $(-5, \infty)$

Find all the critical numbers of the function.

14) $f(x) = 2x^3 + 9x^2 - 24x + 10$ 14) _____

A) -4, 1 B) -1 C) 6 D) 4, -1

15) $f(x) = (x + 2)^{2/5}$ 15) _____

A) 2 B) 5 C) $\frac{4}{5}$ D) -2

16) $y = x^{4/5} - x^{9/5}$ 16) _____

A) $-\frac{4}{5}, 0$ B) $\frac{4}{9}$ C) $0, \frac{4}{9}$ D) $\frac{4}{5}$

Find the open interval(s) where the function is changing as requested.

17) Increasing; $f(x) = x^2 - 2x + 1$ 17) _____

A) $(0, \infty)$ B) $(-\infty, 0)$ C) $(-\infty, 1)$ D) $(1, \infty)$

18) Increasing; $f(x) = \frac{1}{x^2 + 1}$ 18) _____

A) $(-\infty, 0)$ B) $(-\infty, 1)$ C) $(0, \infty)$ D) $(1, \infty)$

19) Decreasing; $f(x) = x^3 - 4x$ 19) _____

A) $\left(-\infty, -\frac{2\sqrt{3}}{3}\right)$ B) $\left(\frac{2\sqrt{3}}{3}, \infty\right)$ C) $(-\infty, \infty)$ D) $\left(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$

20) Increasing; $y = \sqrt{x^2 + 9}$ 20) _____

A) $(0, \infty)$ B) $(-\infty, 0)$ C) $(-1, \infty)$ D) none

Solve the problem.

21) Suppose the total cost C(x) to manufacture a quantity x of insecticide (in hundreds of liters) is given by $C(x) = x^3 - 27x^2 + 240x + 800$. Where is C(x) decreasing? 21) _____

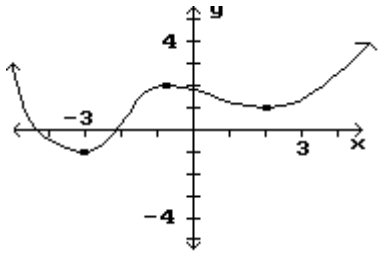
A) (0, 800) B) (8, 800) C) (8, 10) D) (10, 800)

22) Suppose a certain drug is administered to a patient, with the percent of concentration in the bloodstream t hr later given by $K(t) = \frac{6t}{t^2 + 1}$. On what time interval is the concentration of the drug increasing? 22) _____

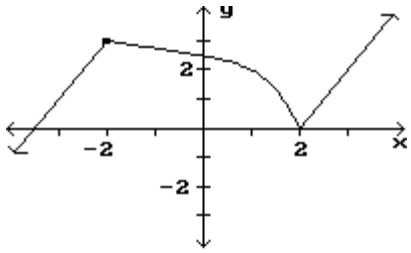
A) (0, 6) B) (6, ∞) C) (0, 1) D) (1, ∞)

Find the location and value of all relative extrema for the function.

23) 23) _____

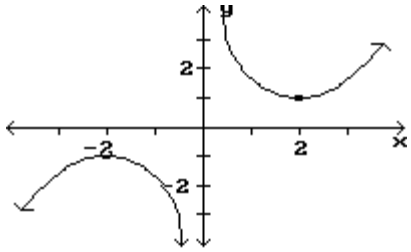


A) Relative minimum of –1 at –3 ; Relative maximum of 2 at –1 ; Relative minimum of 1 at 2.
B) Relative minimum of 0 at –2 ; Relative maximum of –1 at 2 ; Relative minimum of 2 at 1.
C) Relative minimum of –1 at –3 ; Relative maximum of 2 at –1 ; Relative minimum of 0 at 2.
D) Relative minimum of –3 at –1 ; Relative maximum of –1 at 2 ; Relative minimum of 2 at 1.

24) 

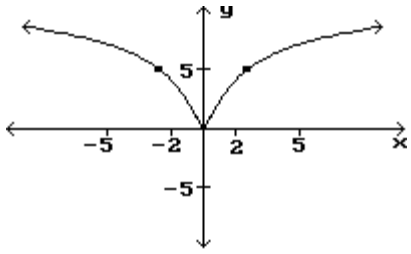
24) _____

A) Relative maximum of 3 at –2.
B) Relative maximum of 3 at –2 ; Relative minimum of 0 at 2.
C) Relative minimum of 0 at 2.
D) None

25) 

25) _____

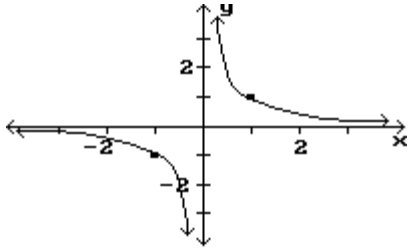
A) None
B) Relative maximum of 2 at 1.
C) Relative minimum of 1 at 2 ; Relative maximum of –1 at –2.
D) Relative minimum of –1 at –2.

26) 

26) _____

A) Relative minimum of 0 at 0.
B) None
C) Relative maximum of 5 at –2 ; Relative maximum of 1 at 2.
D) Relative maximum of 5 at –2 ; Relative minimum of 0 at 0 ; Relative maximum of 1 at 2.

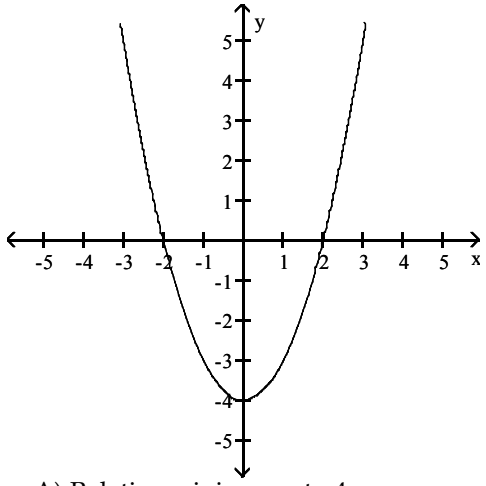
27) 27) _____



A) Relative minimum of 1 at 1.
B) Relative maximum of –1 at –1.
C) Relative maximum of –1 at –1 ; Relative minimum of 1 at 1.
D) None

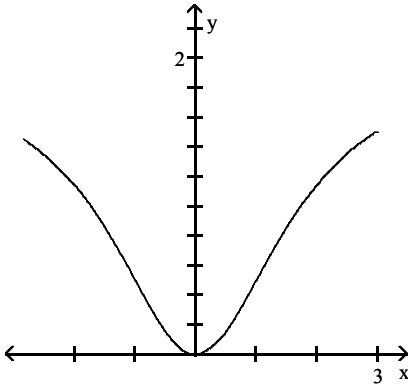
Suppose that the function with the given graph is not f(x), but f′(x). Find the locations of all extrema, and tell whether each extremum is a relative maximum or minimum.

28) 28) _____



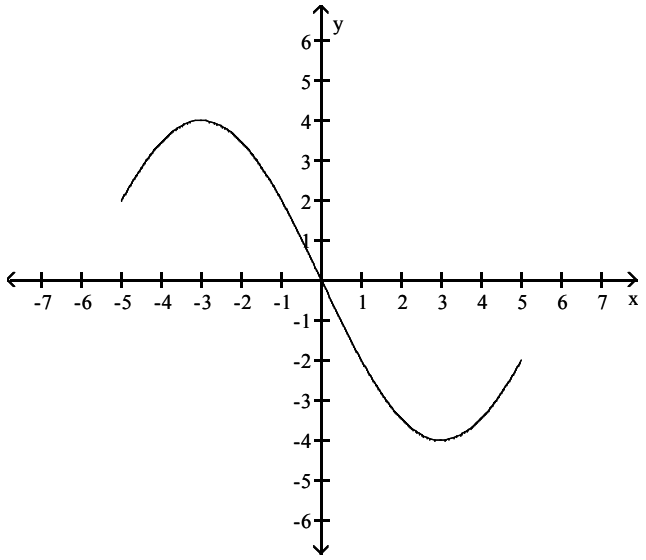
A) Relative minimum at –4
B) Relative maximum at –2; relative minimum at 2
C) Relative maxima at –2 and 2
D) Relative minimum at –2; relative maximum at 2

29) 29) _____



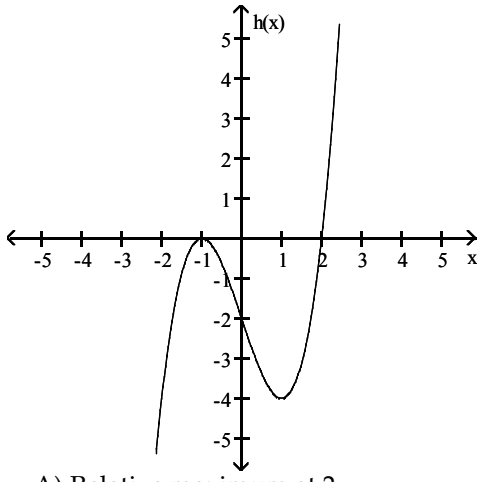
A) Relative minimum at 0
B) Relative maxima at –3 and 3
C) Relative maximum at 0
D) No relative extrema

30) 30) _____



A) Relative minimum at 0
B) Relative maximum at –3; relative minimum at 3
C) No relative extrema
D) Relative maximum at 0

31) 31) _____



A) Relative maximum at 2
B) Relative minima at –1 and 2
C) Relative minimum at 2
D) Relative minimum at 2

Find the x–value of all points where the function has relative extrema. Find the value(s) of any relative extrema.

32) $f(x) = x^2 + 2x - 3$ 32) _____

A) Relative minimum of –2 at 0.
B) Relative maximum of –4 at –1.
C) Relative minimum of 0 at –2.
D) Relative minimum of –4 at –1.

33) $f(x) = x^3 - 3x^2 + 1$ 33) _____

A) Relative maximum of 0 at 1; Relative minimum of –3 at –2.
B) Relative maximum of 1 at 0.
C) No relative extrema.
D) Relative maximum of 1 at 0; Relative minimum of –3 at 2.

34) $f(x) = 3x^4 + 16x^3 + 24x^2 + 32$ 34) _____

A) Relative minimum of 32 at 0.
B) Relative minimum of 30 at –1.
C) Relative maximum of 48 at –2; Relative minimum of 32 at 0.
D) No relative extrema.

35) $f(x) = \frac{1}{x^2 - 1}$ 35) _____

A) Relative maximum of 0 at 1.
B) Relative minimum of –1 at 0.
C) Relative maximum of –1 at 0.
D) No relative extrema.

36) $f(x) = x^{4/3} - x^{2/3}$ 36) _____

A) Relative maximum of 0 at 0; Relative maximum of $-\frac{1}{4}$ at $-\frac{\sqrt{2}}{4}$

B) Relative minimum of of $-\frac{1}{4}$ at $\frac{\sqrt{2}}{4}$

C) Relative maximum of 0 at 0; Relative minimum of $-\frac{1}{4}$ at $\frac{\sqrt{2}}{4}$ and $-\frac{\sqrt{2}}{4}$

D) No relative extrema.

Solve the problem.

37) The annual revenue and cost functions for a manufacturer of grandfather clocks are approximately $R(x) = 480x - 0.03x^2$ and $C(x) = 200x + 100{,}000$, where x denotes the number of clocks made. What is the maximum annual profit? 37) _____

A) $553,333 B) $653,333 C) $853,333 D) $753,333

38) Find the number of units, x, that produces the maximum profit P, if $C(x) = 25 + 56x$ and $p = 100 - 2x$. 38) _____

A) 11 units B) 56 units C) 44 units D) 224 units

39) $S(x) = -x^3 + 6x^2 + 288x + 4000$, $4 \le x \le 20$ is an approximation to the number of salmon swimming upstream to spawn, where x represents the water temperature in degrees Celsius. Find the temperature that produces the maximum number of salmon. 39) _____

A) 4°C B) 8°C C) 12°C D) 20°C

Find f"(x) for the function.

40) $f(x) = 4x^2 + 3x - 8$ 40) _____

A) 0 B) 4 C) 8 D) $8x + 3$

41) $f(x) = 2x^{3/2} - 6x^{1/2}$ 41) _____

A) $3x^{1/2} - 3x^{-1/2}$
B) $1.5x^{1/2} + 1.5x^{-1/2}$
C) $3x^{-1/2} + 3x^{-3/2}$
D) $1.5x^{-1/2} + 1.5x^{-3/2}$

42) $f(x) = x^2 + \sqrt{x}$ 42) _____

A) $\frac{8x^{3/2} + 1}{4x^{3/2}}$ B) $\frac{2x^{3/2} - 1}{x^{3/2}}$ C) $\frac{8x^{3/2} - 1}{4x^{3/2}}$ D) $\frac{2x^{3/2} + 1}{x^{3/2}}$

43) $f(x) = \sqrt{3x - 7}$ 43) _____

A) $\frac{9}{4(3x - 7)^{3/2}}$ B) $-\frac{10}{4(3x - 7)^{3/2}}$ C) $\frac{10}{4(3x - 7)^{3/2}}$ D) $-\frac{9}{4(3x - 7)^{3/2}}$

Find the requested value of the second derivative of the function.

44) $f(x) = x^4 + 4x^3 - 4x + 7$; Find $f''(6)$. 44) _____

A) –575 B) 571 C) 576 D) 580

45) $f(x) = 9x^2 + 9x - 9$; Find $f''(0)$. 45) _____

A) 9 B) 0 C) –18 D) 18

Find the indicated derivative of the function.

46) $f'''(x)$ of $f(x) = 6x^3 + 6x^2 - 6x$ 46) _____

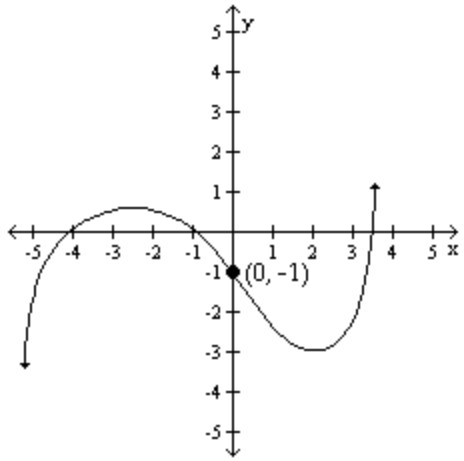
A) 18 B) 18x +36 C) 36 D) 36x + 18

47) $f^{(4)}(x)$ of $f(x) = 2x^6 - 4x^4 + 6x^2$ 47) _____

A) $480x^2 - 48x$ B) $720x^2 - 96x$ C) $720x^2 - 96$ D) $480x^2 - 48$

Find the open intervals where the function is concave upward or concave downward. Find any inflection points.

48) 48) _____



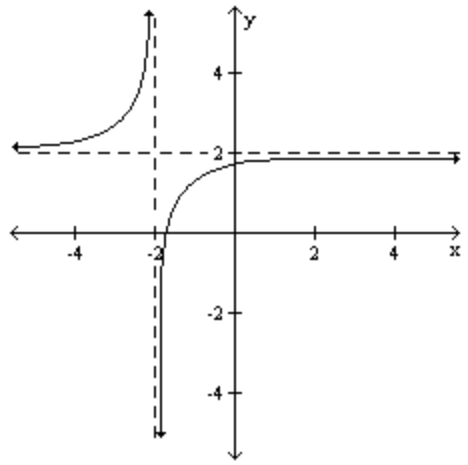
A) Concave upward on (0, ∞); concave downward on (-∞, 0); inflection points at (–4, 0), (–1, 0), and $\left(\frac{7}{2}, 0\right)$

B) Concave upward on (–1, ∞); concave downward on (-∞, 2); inflection point at (2, –3)

C) Concave upward on (0, ∞); concave downward on (-∞, 0); inflection point at (0, –1)

D) Concave upward on (–1, ∞); concave downward on (-∞, 2); inflection points at (–1, 0) and (2, –3)

49) 49) _____



A) Concave upward on $(-\infty, -2)$; concave downward on $(-2, \infty)$; inflection point at $(-2, 2)$
B) Concave upward on $(-2, \infty)$; concave downward on $(\infty, -2)$; inflection point at $(-2, 2)$
C) Concave upward on $(-2, \infty)$; concave downward on $(-\infty, -2)$; no inflection points
D) Concave upward on $(-\infty, -2)$; concave downward on $(-2, \infty)$; no inflection points

Find the largest open intervals where the function is concave upward.

50) $f(x) = x^2 + 2x + 1$ 50) _____

A) $(-1, \infty)$ B) None C) $(-\infty, \infty)$ D) $(-\infty, -1)$

51) $f(x) = 4x^3 - 45x^2 + 150x$ 51) _____

A) $\left(\frac{15}{4}, \infty\right)$ B) $\left(-\frac{15}{4}, \infty\right)$ C) $\left(-\infty, -\frac{15}{4}\right)$ D) $\left(-\infty, \frac{15}{4}\right)$

52) $f(x) = x^3 - 3x^2 - 4x + 5$ 52) _____

A) None B) $(-\infty, 1), (1, \infty)$ C) $(-\infty, 1)$ D) $(1, \infty)$

53) $f(x) = \frac{x}{x^2 + 1}$ 53) _____

A) $(-\infty, -1)$ B) $(-\infty, -1), (-1, \infty)$ C) None D) $(\sqrt{3}, \infty)$

Find any inflection points given the equation.

54) $f(x) = 3x^2 + 12x$ 54) _____

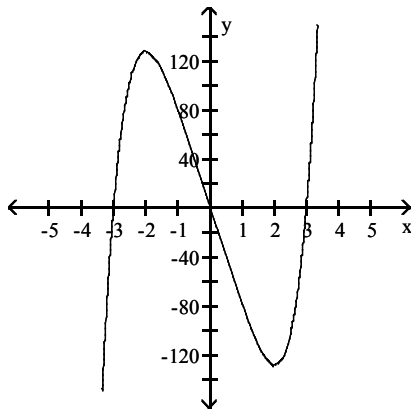
A) No inflection points
B) Inflection point at $(4, -12)$
C) Inflection point at $(-2, -12)$
D) Inflection point at $(-4, -12)$

55) $f(x) = \frac{8x}{x^2 + 4}$ 55) _____

A) Inflection points at $(0, 0), (-2, -2), (2, 2)$
B) No inflection points
C) Inflection points at $(0, 0), \left(-2\sqrt{3}, -2\sqrt{3}\right), \left(2\sqrt{3}, 2\sqrt{3}\right)$
D) Inflection points at $(-2, -2), (2, 2)$

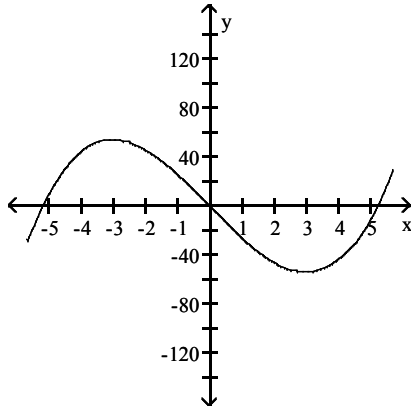
Suppose that the function with the given graph is not f(x), but f′(x). Find the open intervals where the function is concave upward or concave downward, and find the location of any inflection points.

56) 56) _____



A) Concave upward on (-∞, 0); concave downward on (0, ∞); inflection point at 0
B) Concave upward on (-∞, –2) and (2, ∞); concave downward on (–2, 2); inflection points at –120 and 120
C) Concave upward on (-∞, –2) and (2, ∞); concave downward on (–2, 2); inflection points at –2 and 2
D) Concave upward on (–2, 2); concave downward on (-∞, –2) and (2, ∞); inflection points at –2 and 2

57) 57) _____



A) Concave upward on (-∞, 0); concave downward on (0, ∞); inflection point at 0
B) Concave upward on (-∞, –3) and (3, ∞); concave downward on (–3, 3); inflection points at –20 and
20
C) Concave upward on (-∞, –3) and (3, ∞); concave downward on (–3, 3); inflection points at –3 and 3
D) Concave upward on (–3, 3); concave downward on (-∞, –3) and (3, ∞); inflection points at –3 and 3

Decide if the given value of x is a critical number for f, and if so, decide whether the point is a relative minimum, relative maximum, or neither.

58) $f(x) = -x^2 - 16x - 64$; $x = 8$ 58) _____

A) Critical number, relative minimum at (8, –144)
B) Not a critical number
C) Critical number but not an extreme point
D) Critical number, relative maximum at (8, –144)

59) $f(x) = (x^2 - 6)(2x - 3);\ x = \frac{1}{2}$ 59) _____

A) Critical number, relative minimum at $\left(\frac{1}{2}, \frac{23}{2}\right)$

B) Critical number, relative maximum at $\left(\frac{1}{2}, \frac{23}{2}\right)$

C) Not a critical number

D) Critical number but not an extreme point

60) $f(x) = 3x^4 - 4x^3 - 12x^2 + 24;\ x = 0$ 60) _____

A) Critical number but not an extreme point

B) Critical number, relative minimum at (0, 24)

C) Not a critical number

D) Critical number, relative maximum at (0, 24)

61) $f(x) = x^2 - x - 6;\ x = \frac{1}{2}$ 61) _____

A) Critical number, relative minimum at $\left(\frac{1}{2}, -\frac{25}{4}\right)$

B) Critical number but not an extreme point

C) Critical number, relative maximum at $\left(\frac{1}{2}, -\frac{25}{4}\right)$

D) Not a critical number

The function gives the distances (in feet) traveled in time t (in seconds) by a particle. Find the velocity and acceleration at the given time.

62) $s = \frac{1}{t + 3},\ t = 2$ 62) _____

A) $v = -\frac{2}{125}$ ft/s, $a = \frac{1}{25}$ ft/s^2

B) $v = \frac{2}{125}$ ft/s, $a = -\frac{1}{25}$ ft/s^2

C) $v = \frac{1}{25}$ ft/s, $a = -\frac{2}{125}$ ft/s^2

D) $v = -\frac{1}{25}$ ft/s, $a = \frac{2}{125}$ ft/s^2

63) $s = 4t^3 + 2t^2 + 6t + 7,\ t = 2$ 63) _____

A) v = 52 ft/s, a = 62 ft/s^2

B) v = 16 ft/s, a = 32 ft/s^2

C) v = 62 ft/s, a = 52 ft/s^2

D) v = 32 ft/s, a = 16 ft/s^2

64) $s = \sqrt{t^2 - 5},\ t = 3$ 64) _____

A) $v = -\frac{3}{2}$ ft/s, $a = \frac{5}{8}$ ft/s^2

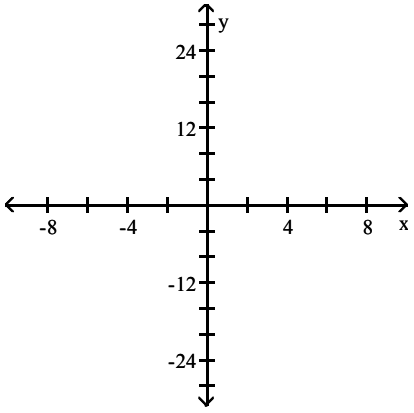
B) $v = -\frac{5}{8}$ ft/s, $a = \frac{3}{2}$ ft/s^2

C) $v = \frac{3}{2}$ ft/s, $a = -\frac{5}{8}$ ft/s^2

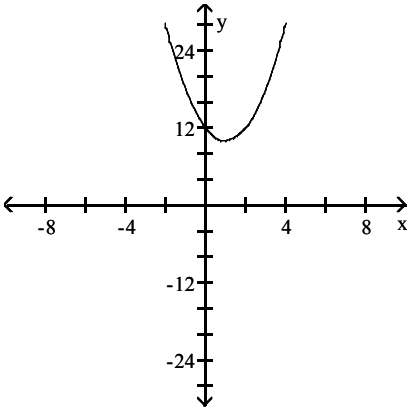
D) $v = \frac{5}{8}$ ft/s, $a = -\frac{3}{2}$ ft/s^2

Sketch the graph and show all extrema, inflection points, and asymptotes where applicable.

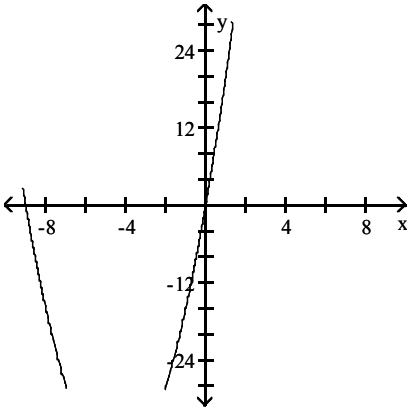
65) $f(x) = 2x^3 + 9x^2 + 12x$ 65) _____



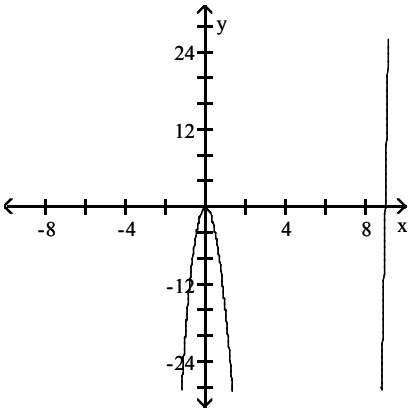
A) Rel min: (1, 10)
No inflection points



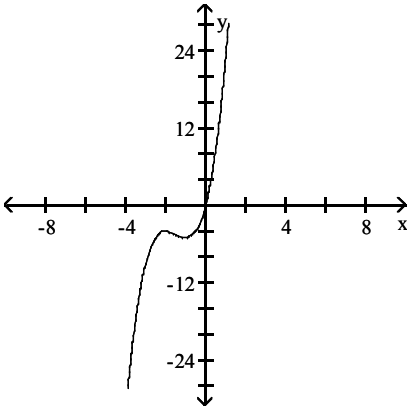
B) No extrema
Inflection point: (0, 0)



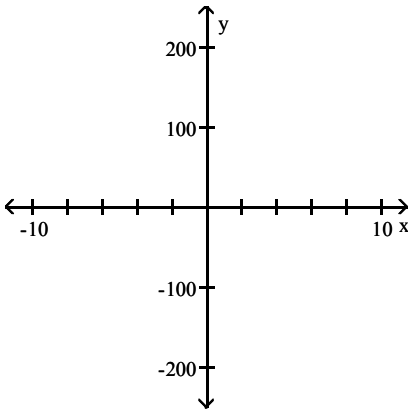
C) Rel max: (0, 0), Rel min: (–6, 216)
Inflection point: (–3, 108)



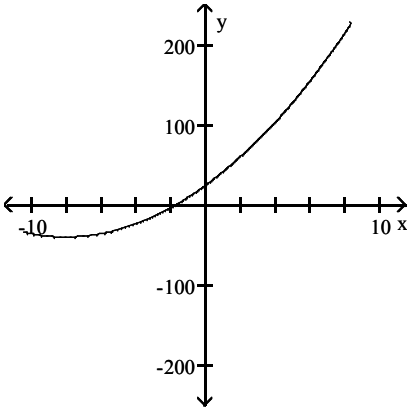
D) Rel max (–2, –4), Rel min: (–1, –5)
Inflection point: $\left(-\frac{3}{2}, -\frac{9}{2}\right)$



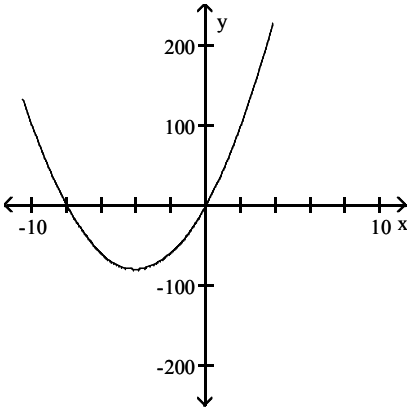
66) $f(x) = 5x^2 + 40x$ 66) _____



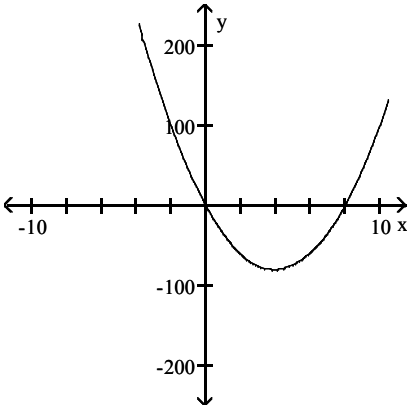
A) Rel min: (–8, –40)
No inflection points



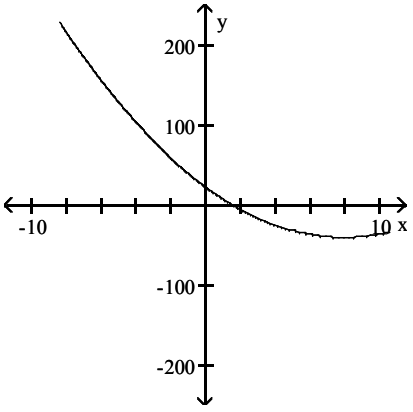
B) Rel min: (–4, –80)
No inflection points



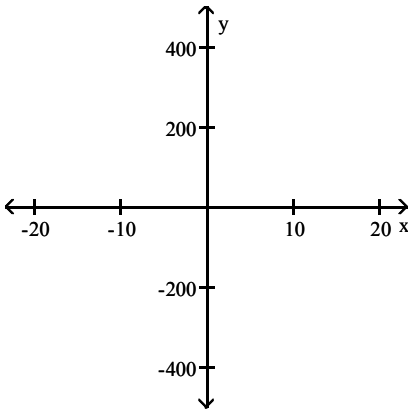
C) Rel min: (4, –80)
No inflection points



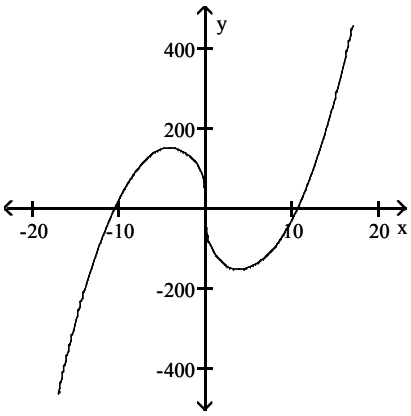
D) Rel min: (8, –40)
No inflection points



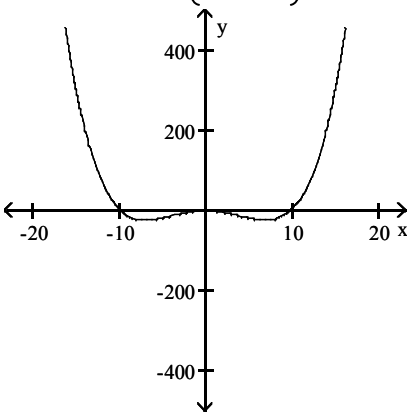
67) $f(x) = x^{1/3}(x^2 - 112)$ 67) _____



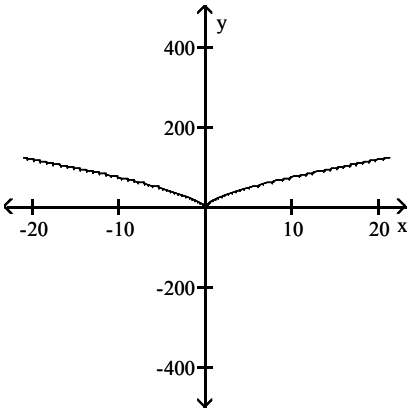
A) Rel max: $(-4, 96\sqrt[3]{4})$, Rel min: $(4, -96\sqrt[3]{4})$
Inflection point: (0,0)



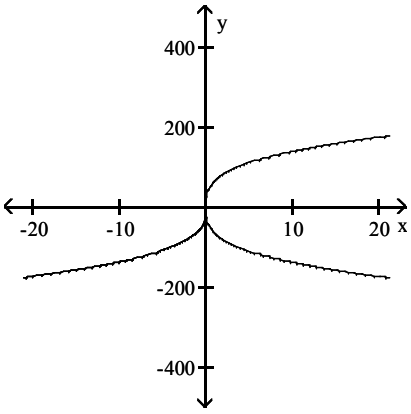
B) Rel max: (0,0), Rel min: $\left(\pm\sqrt{48}, -24\right)$
Inflection point: $\left(\pm 4, -\frac{20}{3}\right)$



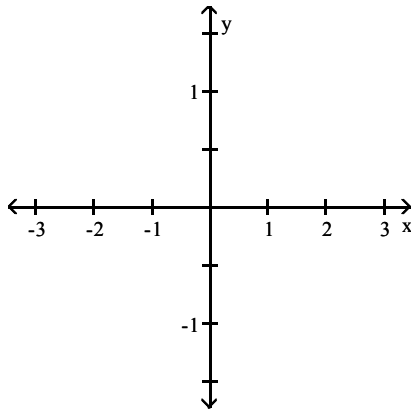
C) Rel min: (0, 0)
No inflection points



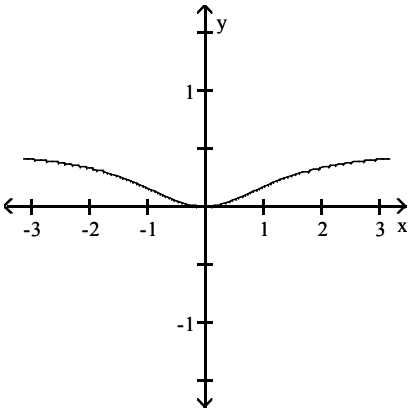
D) No extrema
Inflection point: (0, 0)



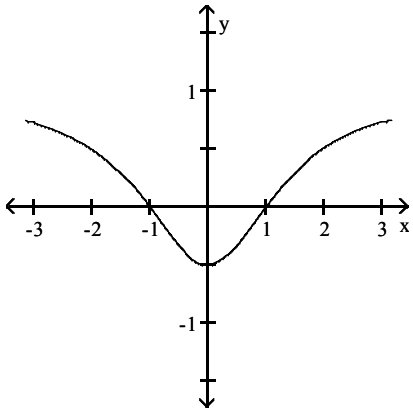
68) $f(x) = \dfrac{x^2}{x^2 + 2}$ 68) _____



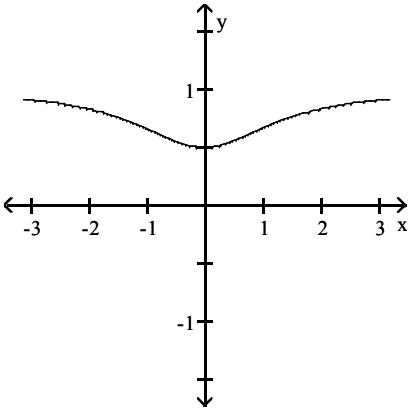
A) Rel min: (0, 0)
No inflection points



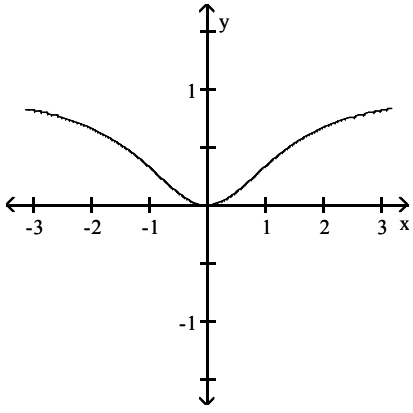
B) Rel min: $\left(0, -\dfrac{1}{2}\right)$
No inflection points



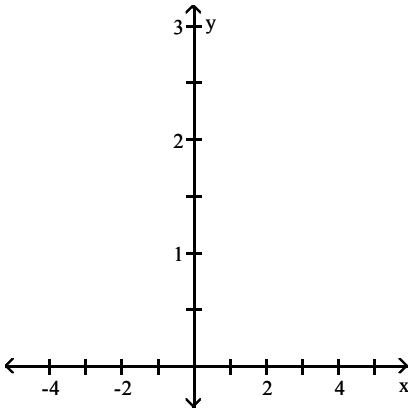
C) Rel min: $\left(0, \dfrac{1}{2}\right)$
No inflection points



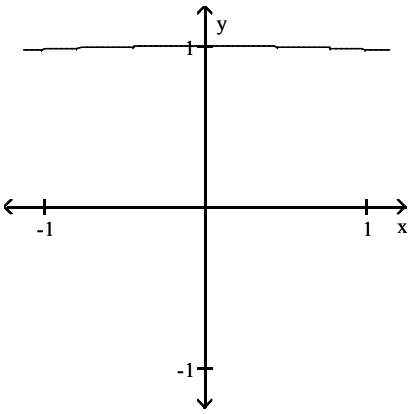
D) Rel min: (0, 0)
Inflection points: $\left(-\dfrac{\sqrt{6}}{3}, \dfrac{1}{4}\right), \left(\dfrac{\sqrt{6}}{3}, \dfrac{1}{4}\right)$



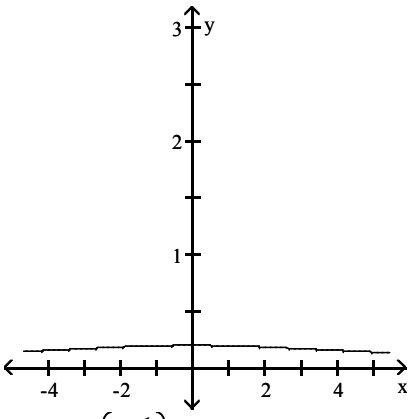
69) $f(x) = \dfrac{1}{\sqrt{25 - x^2}}$ 69) _____



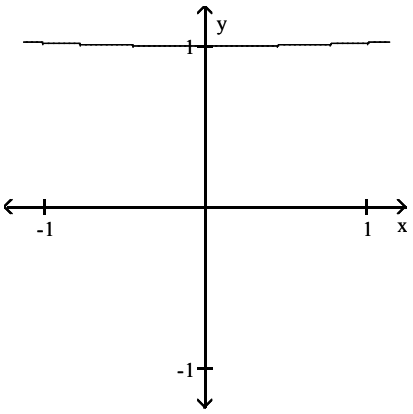
A) Rel max: (0, 1)
No inflection points



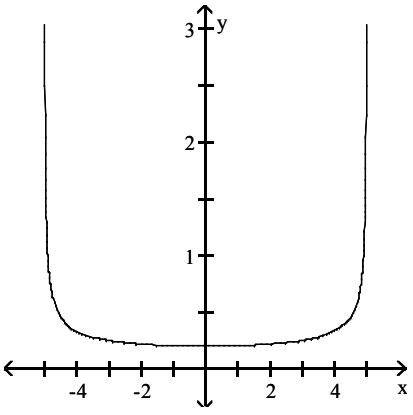
B) Rel max: $\left(0, \dfrac{1}{5}\right)$
No inflection points



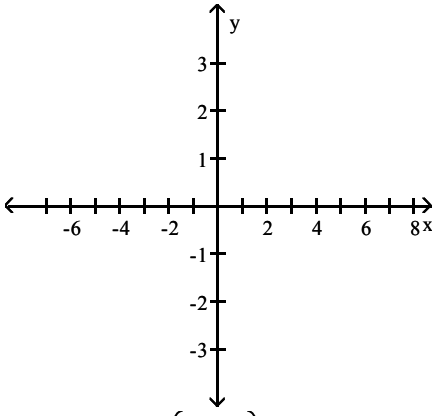
C) Rel min: (0, 1)
No inflection points



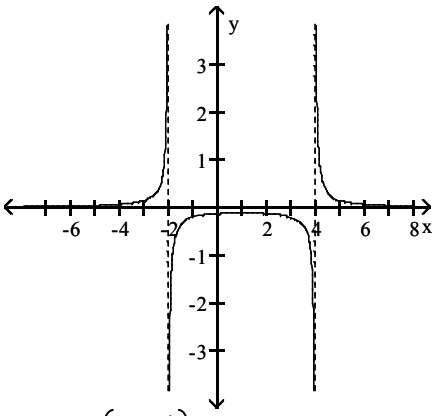
D) Rel min: $\left(0, \dfrac{1}{5}\right)$
No inflection points



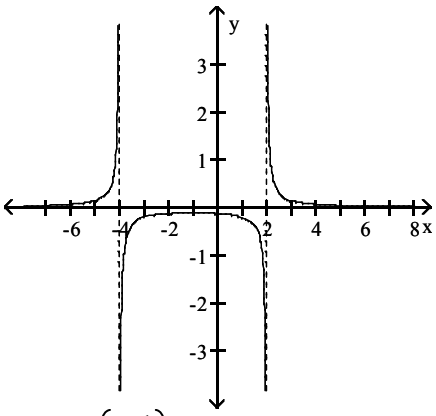
70) $f(x) = \dfrac{1}{x^2 + 2x - 8}$ 70) _____



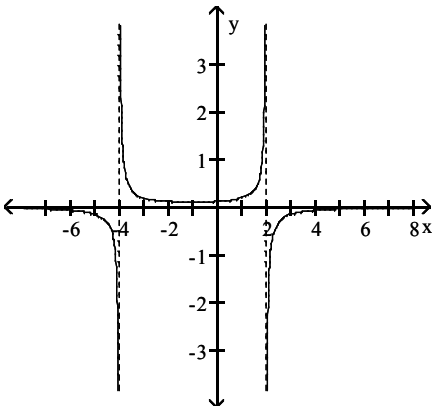
A) Rel max: $\left(1, -\dfrac{1}{9}\right)$
No inflection points



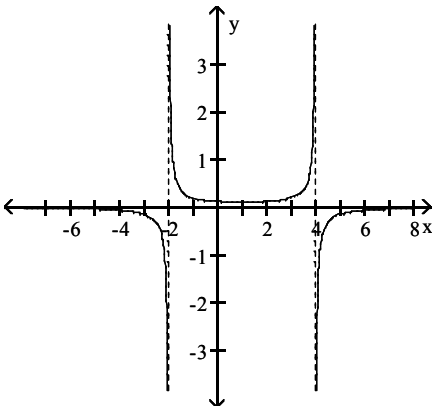
B) Rel max: $\left(-1, -\dfrac{1}{9}\right)$
No inflection points



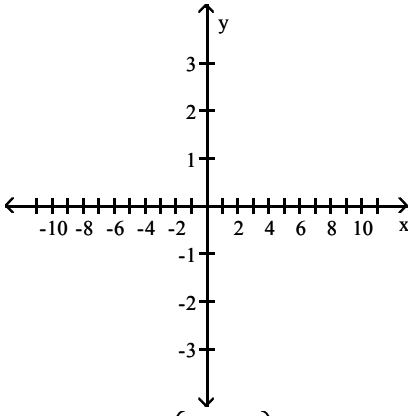
C) Rel min: $\left(-1, \dfrac{1}{9}\right)$
No inflection points



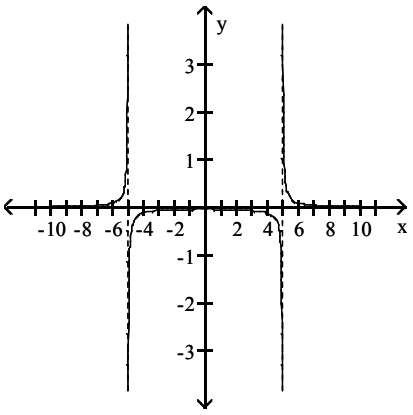
D) Rel min: $\left(1, \dfrac{1}{9}\right)$
No inflection points



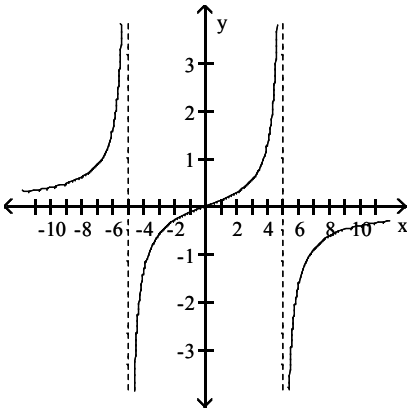
71) $f(x) = \dfrac{3x}{x^2 - 25}$ 71) _____



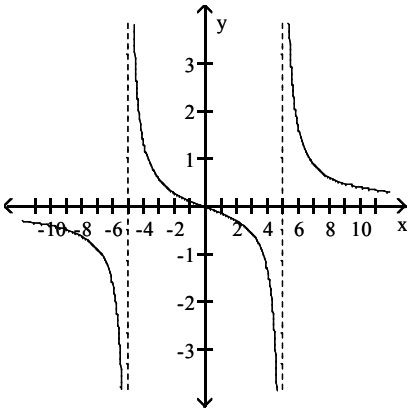
A) Rel max: $\left(0, -\dfrac{1}{25}\right)$
No inflection points



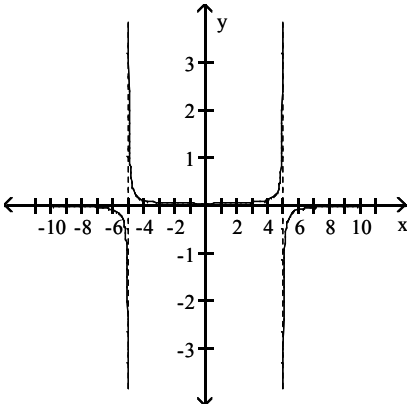
B) No extrema
Inflection point: (0, 0)



C) No extrema
Inflection point: (0, 0)

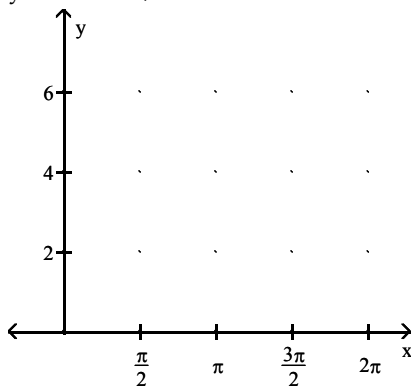


D) Rel min: $\left(0, \dfrac{1}{25}\right)$
No inflection points



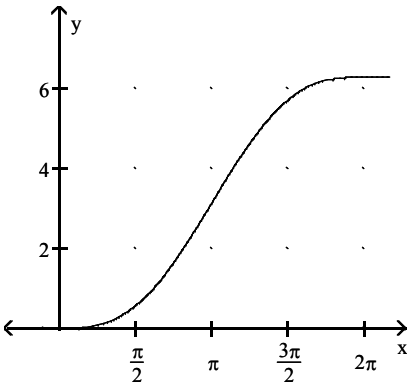
Sketch the graph and show all local extrema and inflection points.

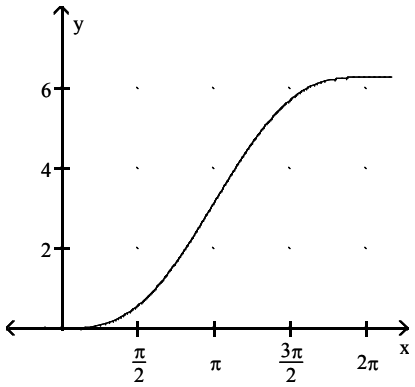
72) $y = x + \sin x,\ 0 \le x \le 2\pi$ 72) _____



A) Local minimum: (0, 0)
Local maximum: $(2\pi, 2\pi)$
No inflection points

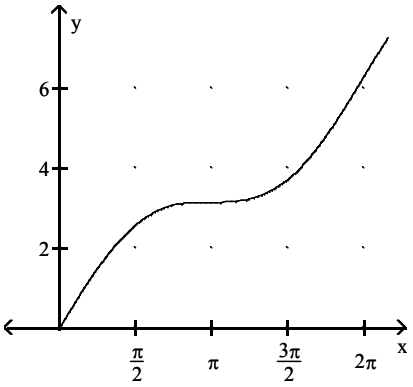
B) Local minimum: (0, 0)
Local maximum: $(2\pi, 2\pi)$
Inflection point: (π, π)

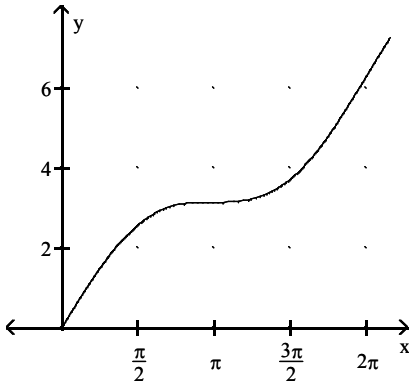




C) Local minimum: (0, 0)
Local maximum: $(2\pi, 2\pi)$
No inflection points

D) Local minimum: (0, 0)
Local maximum: $(2\pi, 2\pi)$
Inflection point: (π, π)





Solve the problem.

73) The percent of concentration of a certain drug in the bloodstream x hours after the drug is administered is given by $K(x) = \frac{5x}{x^2 + 9}$. At what time is the concentration a maximum? 73) _____

A) 0.5 hr B) 5 hr C) 3 hr D) 0.9 hr

74) Find the point of diminishing returns (x, y) for the function $R(x) = 3000 - x^3 + 39x^2 + 400x$, $0 \le x \le 20$, where R(x) represents revenue in thousands of dollars and x represents the amount spent on advertising in tens of thousands of dollars. 74) _____

A) (14 , 13,500)
B) (15.6, 14,934.62)
C) (56.39, –29,741.28)
D) (13 , 12,594)

Answer Key
Testname: CHAPTER 3 (PART 1) 1ST AND 2ND DER ANALYSIS, CURVE SKETCHING

1) A
2) C
3) B
4) B
5) C
6) D
7) D
8) C
9) C
10) A
11) B
12) B
13) A
14) A
15) D
16) C
17) D
18) A
19) D
20) A
21) C
22) C
23) A
24) B
25) C
26) A
27) D
28) B
29) D
30) D
31) D
32) D
33) D
34) A
35) C
36) C
37) A
38) A
39) C
40) C
41) D
42) C
43) D
44) C
45) D
46) C
47) C
48) C
49) D
50) C

51) A
52) D
53) D
54) A
55) C
56) C
57) C
58) B
59) C
60) D
61) A
62) D
63) C
64) C
65) D
66) B
67) A
68) D
69) D
70) B
71) C
72) D
73) C
74) D